

DEVELOPMENT OF NON-EUCLIDEAN GEOMETRY

The following problems are stated for the hyperbolic (Lobachevskian) plane.

Problem 1. Prove that, for any pair of intersecting lines a , b , there is a line c perpendicular to a and disjoint from b .

Problem 2. In the preceding problem, can the assumption that a and b intersect be relaxed?

Problem 3. For any angle Θ and a point A inside Θ , there exists a line through A not intersecting the boundary rays of Θ . Prove or disprove this statement.

Problem 4. The areas of all triangles are uniformly bounded by the same number. Prove or disprove this statement. The same problem for quadrilaterals.

The following problems are stated for the Euclidean 3-space. Let S_R denote the sphere in the 3-space centered at the origin and having radius R . In other words, S_R is given in coordinates x, y, z by the equation

$$x^2 + y^2 + z^2 = R^2.$$

Define *great circles* in S_R as circles in S_R , whose centers coincide with the origin. A *triangle* in S_R is defined as a part of S_R bounded by three arcs of great circles.

Problem 5. The sum of angles of any triangle in S_R is greater than π . Prove or disprove.

Problem 6. Prove that the angular defect of any triangle in S_R is proportional to its area. Find the coefficient of proportionality.